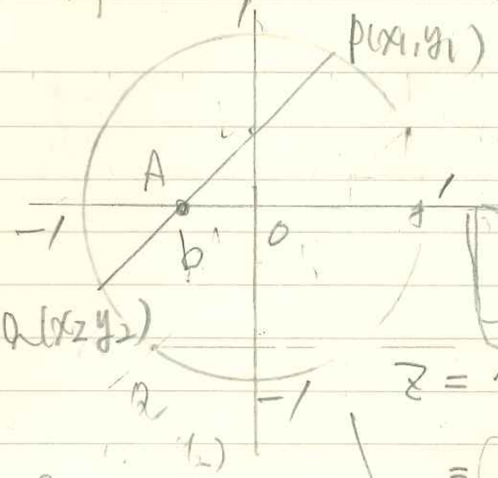
No.

DATE 3.14 ~ 3.15

問9.



$PA^2 + QA^2 = (x_1 - x)^2 + (y_1 - y)^2 + (x_1 - x_2)^2 + (y - y_2)^2$

$\boxed{y = a(x - b)}$

$z = x_1^2 + (y_1 + b)^2 + (y_2 - b)^2 + x_2^2$

$= 1 + 2by_1 + b^2 + b^2 - 2by_2 + 1$

$y_1 = ax_1 - b \quad y_2 = ax_2 - b$

$= 1 + 2abx_1 - 2b^2 + b^2 + b^2 - 2abx_2 + 2b^2 + 1$

$\cancel{= 2b^2 + 2} - 2ab(x_1 + x_2)$

$PA^2 + QA^2 = (x_1 - b)^2 + y_1^2 + (x_2 - b)^2 + y_2^2$

$= (x_1^2 + y_1^2) - 2bx_1 + b^2 + x_2^2 + y_2^2 - 2bx_2 + b^2$

$= 2b^2 - 2b(x_1 + x_2) + 2$

$= 2\{b^2 - 2b(x_1 + x_2) + 1\}$

$y_1 = a(x_1 - b) \quad y_2 = a(x_2 - b)$

$x_1^2 + y_1^2 = 1 \quad x_1^2 + a^2(x_1^2 - 2bx_1 + b^2) = 1$

$y = a(x - b)$

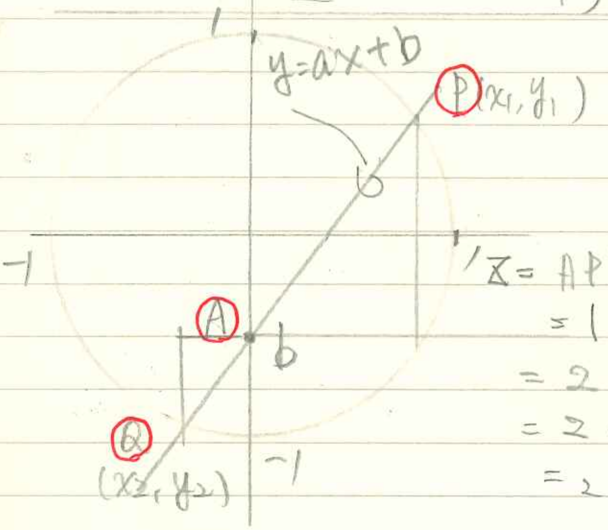
$x^2 + y^2 = \phi$

$(a^2+1)x_1^2 - 2ba^2x_1 + a^2b^2 - 1 = 0$

$x_1 = \dfrac{2a^2b \pm \sqrt{4a^4b^2 - 4(a^2+1)(a^2b^2-1)}}{2(a^2+1)}$

$4(a^4b^2 - a^2) + (a^2b^2 - 1)$

$= \dfrac{2a^2b \pm \sqrt{a^2(b^2-1) - 1}}{2(a^2+1)}$



$x_1^2 + y_1^2 = 1$ —①

$y_1 = ax_1 + b$ —②

$x_2^2 + y_2^2 = 1$ —③

$y_2 = ax_2 + b$ —④

(値をaを変数として Zの極小をねらう, bは定数)

$Z = AP^2 + AQ^2 = x_1^2 + (y_1 - b)^2 + x_2^2 + (y_2 - b)^2$

$= 1 - 2by_1 + b^2 + 1 - 2by_2 + b^2$

$= 2\{1 + b^2 - 2b(y_1 + y_2)\}$

$= 2\{1 + b^2 - 2b(a(x_1 + x_2) + 2b)\}$

$= 2\{-2ab(x_1 + x_2) - 3b^2 + 1\}$

①②より $x_1^2 + a^2x_1^2 + 2abx_1 + b^2 = 1 \quad (a^2+1)x^2 + 2abx_1 + (b^2 - 1) = 0$

$x_1 = \dfrac{-2ab \pm \sqrt{4a^2b^2 - 4(a^2+1)(b^2-1)}}{2(a^2+1)} = \dfrac{-ab \pm \sqrt{(a^2+1) - b^2}}{a^2+1}$

$4(a^2b^2 + b^2 - a^2 - 1)$

$x_1 + x_2 = \dfrac{-2ab}{a^2+1}$

$z = 2\left\{\dfrac{+4a^2b^2}{a^2+1} - 3b^2 + 1\right\}$

$$z' = 2 \cdot \frac{8ab^2(a^2+1) - 4a^2b^2 \times 2a}{(a^2+1)^2} = \frac{16ab^2}{(a^2+1)^2}$$

a	$a<0$	0	$a>0$
z'	$z'<0$		$z'>0$

傾き a が 0 の時に $z = PA^2 + QA^2$ は極小になる。従って直線が $y = b$ になる時であり、$PA = QA$ の時である。